T-dual R-R zero-norm states, D-branes and S-duality of type II string theory

Jen-Chi Lee*

Department of Electrophysics, National Chiao Tung University, Hsinchu 30050, Taiwan, R. O. C.

Abstract

We calculate the R-R zero-norm states of type II string spectrum. To fit these states into the right symmetry charge parameters of the gauge transformations of the R-R tensor forms, one is forced to T-dualize some type I open string space-time coordinates and thus to introduce D-branes into the theory. We also demonstrate that the constant T-dual R-R 0-form zero-norm state, together with the NS-NS singlet zero-norm state are responsible for the SL(2,Z) S-duality symmetry of the type II B string theory.

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*e-mail: jcclee@cc.nctu.edu.tw

I. INTRODUCTION

It has been pointed out for a long time that the complete space-time symmetry [1] of string theory is related to the zero-norm state (a physical state that is orthogonal to all physical states including itself) in the old covariant quantization of the string spectrum. [2] This observation had made it possible to explicitly construct many stringy ($\alpha' \to \infty$) massive symmetry of the theory. This includes the w_∞ symmetry of the toy 2D string [3] and the *discrete* massless and massive T-duality symmetry of closed bosonic string. [4] The authors of [5] show that, in string theory, some target space mirror symmetry of N=2 backgrounds on group manifolds is a Kac-Moody gaugy symmetry. Thus, like T-duality, it should be related to the zero-norm states. On the other hand, the massless and massive SUSY, and some new enlarged spacetime boson-fermion symmetries induced by zero-norm states were also discussed in [6]. It is thus of interest to study the R-R zero-norm state and its relation to D-brane which was recently shown by Polchinski to be the symmetry charge carrier of the propagating R-R forms. [7]

Presumably, there should be no R-R zero-norm state in the type II string spectrum since the fundamental string does not interact with the R-R forms. However, to our surprise, it was discovered that there do exist both massless and massive R-R zero-norm states in the type II string spectrum. [6] It was then realized that the degree of freedom of massless R-R zero-norm states does not fit into that of the symmetry parameters of the propagating R-R forms and thus resolved the seeming inconsistency. This observation gives us another justification of the well-known wisdom that perturbative string does not carry the massless R-R charges, although the existence of these R-R zero-norm states remain mysterious.

In this paper, we will show that the T-dual R-R zero-norm states serve as the right symmetry parameters of the gauge transformations of the R-R propagating forms. Also one is forced to introduce Type I open string and D-branes into the type II string theory to incorporate these T-dual R-R zero-norm states. Our *spacetime* zero-norm states argument here is in complementary with the *worldsheet* string vertex operator argument first given

by Binnchi, Pradisi and Sagnotti. [8] They considered R-R one-point function in the (-$\frac{1}{2}$,-$\frac{3}{2}$) ghost picture on the *disk* and resulted in a conclusion which was consistent with D-brane as R-R charge carrier [7]. As an important application, we demonstrate that the constant T-dual R-R 0-form zero-norm state, together with the NS-NS singlet zero-norm state which was always neglected in the previous discussions, are responsible for the *discrete* SL(2,Z) S-duality symmetry of the type II B string theory. [9] This discovery suggests that not only stringy ($\alpha' \to \infty$) symmetry but also strong-weak ($g_s \to \infty$) duality symmetry are related to the existence of zero-norm states of the spectrum.

II. T-DUAL R-R ZERO-NORM STATES

The massless physical NS state of the open superstring is (we use the notation in Ref [10])

$$\varepsilon_\mu b^\mu_{-\frac{1}{2}} \left|0,k\right\rangle ; \;\; k \cdot \varepsilon = 0, \;\; k^2 = 0 \tag{1}$$

In addition, there is a singlet zero-norm state

$$k_\mu b^\mu_{-\frac{1}{2}} \left|0,k\right\rangle ; \;\; k^2 = 0 \; . \tag{2}$$

The NS-NS symmetries of graviton and antisymmetry tensor of type II string were derived through the following two zero-norm states

$$\begin{aligned} &\varepsilon_\mu b^\mu_{-\frac{1}{2}} \left|0,k\right\rangle \otimes k_\mu \tilde{b}{}^{\mu}_{-\frac{1}{2}} \left|0,k\right\rangle , \\ &k_\mu b^\mu_{-\frac{1}{2}} \left|0,k\right\rangle \otimes \varepsilon_\mu \tilde{b}{}^{\mu}_{-\frac{1}{2}} \left|0,k\right\rangle \end{aligned} \tag{3}$$

in the first order weak field approximation (WFA) [11]. The remaining interesting singlet zero-norm state

$$k_\mu b^\mu_{-\frac{1}{2}} \left|0,k\right\rangle \otimes k_\mu \tilde{b}{}^{\mu}_{-\frac{1}{2}} \left|0,k\right\rangle \tag{4}$$

will be discussed in the next section.

We now discuss the massless R state. The only propagating spinor is

$$\left|\overrightarrow{S},k\right\rangle u_{\overrightarrow{s}};\ F_0\left|\overrightarrow{S},k\right\rangle u_{\overrightarrow{s}}=0\ . \tag{5}$$

The GSO operator in the massless limit reduces to the chirality operator, and only one of the chiral spinor 8_s(or 8_c) will be projected out. In addition, there is a massless fermionic zero-norm state

$$k_\mu\Gamma^\mu_{\overrightarrow{s},\overrightarrow{s}}\left|\overrightarrow{S},k\right\rangle\theta_{\overrightarrow{s}}\ . \tag{6}$$

Eq.(6) is the only massless solution of the following

$$F_0\left|\psi\right\rangle,\text{where } F_1\left|\psi\right\rangle=L_0\left|\psi\right\rangle=0. \tag{7}$$

The state in equation (6) is crucial in the discussion of this paper. Note that $k\cdot\Gamma\left|\overrightarrow{S},k\right\rangle\theta_{\overrightarrow{s}}$ is left-handed if $\left|\overrightarrow{S},k\right\rangle\theta_{\overrightarrow{s}}$ is right-handed and both spinors have exactly the same degree of freedom. The massless propagating R-R states of type II string consist of tensor forms

$$G_{\alpha\beta}=\sum_{k=0}^{10}\frac{i^k}{k!}G_{\mu_1\mu_2\ldots\mu_k}(\Gamma^{\mu_1\mu_2\ldots\mu_k})_{\alpha\beta}, \tag{8}$$

where $\Gamma^{\mu_1\mu_2\ldots\mu_k}$ are the antisymmetric products of gamma-matrix, and α,β are spinor indices. There is a duality relation which reduces the number of independent tensor components to up to k=5 form. The on-shell conditions, or two massless Dirac equations, imply G is indeed a field strength and can be written as

$$G_{(k)}=dA_{(k-1)} \tag{9}$$

which means perturbative string states do not carry the *massless* R-R symmetry charges. We are now in a position to discuss the R-related symmetry charges. Let's first introduce the NS-R (R-NS) SUSY zero-norm states [6]

$$k\cdot b_{-\frac{1}{2}}\left|0,k\right\rangle\otimes\left|\overrightarrow{S},k\right\rangle\overline{u}_{\overrightarrow{s}}\text{and }\left|\overrightarrow{S},k\right\rangle u_{\overrightarrow{s}}\otimes k\cdot\widetilde{b}_{-\frac{1}{2}}\left|0,k\right\rangle \tag{10}$$

for the II A theory and a trivial modification for the II B theory. The corresponding worldsheet vertex operator in the $(0,-\frac{1}{2})$ picture for say the first state in equation (10) is

$$k_\mu(\partial x^\mu(z) + ik\cdot\psi\psi^\mu)e^{ik\cdot x(z)}u_\alpha \tilde{S}^\alpha (\overline{z})e^{-\frac{1}{2}\overset{\sim}{\phi}}e^{ik\cdot x(\overline{z})}$$
$$= \partial e^{ik\cdot x(z)}u_\alpha \tilde{S}^\alpha e^{-\frac{1}{2}\overset{\sim}{\phi}}e^{ik\cdot x(\overline{z})}, \tag{11}$$

which is a worldsheet total derivative and, as in the case of bosonic sector [2], one can introduce a worldsheet generator and deduce the SUSY current to be

$$\tilde{Q}_{\alpha,-\frac{1}{2}} = \tilde{S}^\alpha e^{-\frac{1}{2}\overset{\sim}{\phi}}, \tag{12}$$

where $\tilde{S}^\alpha$ and $\overset{\sim}{\phi}$ are the right-moving spin field and the bosonized superconformal ghost respectively. This *zero-norm state derivation* is consistent with the original approach. [12] The advantage of our approach is that one can generalize to derive the enlarged stringy boson-fermion symmetry by using the massive fermion zero-norm state of the spectrum. We give one example here. There exists a m=2 NS-R zero-norm state

$$\left[2\theta_{\mu\nu}\alpha^\mu_{-1}b^\nu_{-\frac{1}{2}} + k_{[\lambda}\theta_{\mu\nu]}b^\lambda_{-\frac{1}{2}}b^\mu_{-\frac{1}{2}}b^\nu_{-\frac{1}{2}}\right]|0,k\rangle \otimes \widetilde{\alpha}^\lambda_{-1}\left|\overrightarrow{S},k\right\rangle u_{\lambda,\overrightarrow{s}} \tag{13}$$

with $\theta_{\mu\nu} = -\theta_{\nu\mu}$, $k^\mu\theta_{\mu\nu} = 0$ and

$$[(k\cdot d_0)\alpha^\mu_{-1} + d^\mu_{-1}]u_{\mu,\overrightarrow{s}} = 0,$$
$$d^\mu_0 u_{\mu,\overrightarrow{s}} = 0. \tag{14}$$

The corresponding vertex operator is calculated to be

$$[2\theta_{[\mu\nu],\lambda\alpha}(\partial x^\mu\partial x^\nu - \psi^\mu\partial\psi^\nu + ik\cdot\psi\psi^\mu\partial x^\nu) + k_{[\delta}\theta_{\mu\nu],\lambda\alpha}(3\partial x^\mu + ik\cdot\psi\psi^\mu)$$
$$\psi^\nu\psi^\delta]\overline{\partial}x^\lambda k\cdot\overline{\psi}e^{-\frac{1}{2}\overset{\sim}{\phi}}\tilde{S}^\alpha e^{ik\cdot x(z,\overline{z})} \tag{15}$$

where $\theta_{\mu\nu,\lambda\alpha} \equiv \theta_{\mu\nu}\cdot u_{\lambda\alpha}$. It is straight-forward to construct the corresponding ward identity although the symmetry transformation law of the background fields is not easy to write down at this point.

We now turn to discuss the R-R zero-norm states. For the massless level, we have the following zero-norm states

$$k_\mu\Gamma^\mu_{\overrightarrow{s}'\overrightarrow{s}}\left|\overrightarrow{S},k\right\rangle\theta_{\overrightarrow{s}}\otimes\left|\overrightarrow{S},k\right\rangle u_{\overrightarrow{s}} \qquad \text{(II A)} \tag{16}$$

and

$$k_\mu \Gamma^\mu_{\vec{s}'\vec{s}} \left|\vec{S},k\right\rangle \overline{\theta_{\vec{s}}} \otimes \left|\vec{S},k\right\rangle u_{\vec{s}} \qquad \text{(II B)}. \tag{17}$$

These are tensor forms as in equation (8). The on-shell condition on the right mover together with the trivial identity $(k\cdot\Gamma)^2 \left|\vec{S},k\right\rangle \theta_{\vec{s}} = 0$ on the left mover imply, as in equation (9), that

$$F_{(k)} = d\omega_{(k-1)}. \tag{18}$$

Note that, for the II A (II B) theory, $\omega_{(p)}$ in eq(18) does not fit into the gauge symmetry parameters of $A_{(p)}$ forms of II A (II B) theory in eq(9) since they share the same tensor index structures. In fact, for a $p+1$ form $A_{(p+1)}$, one needs a p form $\widetilde{\omega_{(p)}}$ symmetry parameters, as can be seen from its spacetime coupling to D-brane

$$\int_{\text{world vol of D-brane}} A_{(p+1)} \equiv \int d^{p+1}\xi A_{\mu_1\mu_2\cdots\mu_{p+1}}(x)\partial_1 x^{\mu_1}\cdots\partial_{p+1}x^{\mu_{p+1}}, \tag{19}$$

which implies a space-time gauge symmetry

$$A_{(p+1)} \to A_{(p+1)} + d\widetilde{\omega}_{(p)}. \tag{20}$$

This justifies that no perturbative type II string state carries the R-R charge. On the other hand, it is well-known that each time we T-dualize in an additional direction the dimension of the D-branes goes down by one and the R-R forms lose an index. To include the right closed string zero-norm state $\widetilde{\omega}_{(p)}$, one is thus forced to introduce the type I *unoriented* open string and T-dualizes k = odd (even) numbers of space-time coordinates and then takes the noncompact limit $R \to 0$ for each compatified radius. For k = odd (even), one has type II A (II B) string states *in the bulk far away from D-branes.* The right $\widetilde{\omega}_{(p)} \equiv \omega^{(T)}_{(p+1)}$ state, the T-dual R-R zero-norm state, is thus attached to the D p-brane for p= even (odd) in II A (II B) theory. Note that, near the D-branes, the orientation projection of the Type I theory leaves only one linear combination of two SUSY charges(SUSY zero-norm states in eq.(10)) of the Type II theory in the bulk. It is $Q'_\alpha + (\Pi^k_m \beta^m \widetilde{Q}')_\alpha$ with $\beta^m \equiv \Gamma^m\Gamma$. The

T-dual R-R zero-norm states attached in the boundary of the open string 1-loop diagram with D-branes are the T-dual version of the R-R zero-norm states in the bulk of the closed string tree diagram. Our argument resolves the puzzle of seeming unwanted R-R zero-norm states in perturbative type II string spectrum and simultaneously *motivates the introduction of D-branes into the theory* which is complementary to the argument in Ref [7]. The space-time T-dual R-R zero-norm state has an interesting analogy from worldsheet vertex operator point of view. The authors of [8] considered one point function of R-R vertex operator on the *disk*. Since the total right + left ghost charge number must add up to -2, one is forced to change the vertex operator in the conventional $(-\frac{1}{2}, -\frac{1}{2})$ picture to either $(-\frac{1}{2}, -\frac{3}{2})$ or $(-\frac{3}{2}, -\frac{1}{2})$ picture. This inverse picture changing involves, among other unrelated things, a factor of $k \cdot \Gamma$, which shifts the field strength to the potential and gives a strong hint that D-brane carries the R-R charge. On the other hand, our space-time T-dual R-R zero-norm states do contain this important $k \cdot \Gamma$ factor as can be seen from eqs (16) and (17). This again gives a strong support of our space-time T-dual R-R zero-norm state approach. In the next section, we will see an even more interesting application of these states.

III. DILATON-AXION SYMMETRIES AND SL(2,Z) S-DUALITY

According to section II, for II A theory, we have $A_{(1)}$, $A_{(3)}$ potentials with $d\omega_{(0)}^{(T)}$, $d\omega_{(2)}^{(T)}$ T-dual zero-norm states and, for II B theory, we have $A_{(2)}$, $A_{(4)}$ potential with $d\omega_{(1)}^{(T)}$, $d\omega_{(3)}^{(T)}$ T-dual zero-norm states for their symmetry charge parameters. For completeness we have, in addition, an axion $A_{(0)} \equiv \chi$ in the II B theory. The corresponding T-dual zero-norm state is naturally identified to be the constant 0-form $F_{(0)}^{(T)}$, which is Poincare dual to the *constant* 10-form $F_{(10)}^{(T)} \equiv d\omega_{(9)}^{(T)}$. So we have the "symmetry"

$$\chi \rightarrow \chi + F_{(0)}^{(T)}. \tag{21}$$

Note that, in eq. (8), there is a constant 10-form field strength $G_{(10)} = dA_{(9)}$ which is Poincare dual to the constant 0-form field strength in II A theory as well. This non-propagating degree of freedom can be included in the massive type II A supergravity and

was conjectured to be related to the cosmological constant. See the interesting discussion of this 9-form potential $A_{(9)}$ by Polchinski in Ref. [7]. Equation (21) is consistent with the fact that the axion χ is defined up to a constant. The interesting new result here is that we naturally identify this constant to be $F^{(T)}_{(0)}$.

We now turn to the discussion of NS-NS dilaton ϕ. Remember we have a Remaining NS-NS singlet zero-norm state in equation (4). The physical meaning of this state will be discussed in the following. In reference [11] each space-time symmetry of the bosonic background field in the first order WFA can be constructed through a superconformal deformation

$$(T^{(1)} = \overline{T}^{(1)}, T_F^{(1)}, \overline{T}_F^{(1)}) \tag{22}$$

corresponding to a spacetime zero-norm state.

In equation (22), $T^{(1)}(T_F^{(1)})$ is the upper component (lower component) of deformation of the superstress tensor in the first order WFA when the background field is turned on. $\overline{T}^{(1)}, (\overline{T}_F^{(1)})$ is its anti-holomorphic counterpart. It was shown that superconformal deformations constructed from zero-norm states in eq.(3) give the symmetries of graviton and antisymmetric tensor. The superconformal deformation constructed from the zero-norm state in eq.(4), which was neglected in the previous discussion., is calculated to be

$$\begin{aligned} T^{(1)} = \overline{T}^{(1)} &= \partial_\mu\partial_\nu\theta(\partial x^\mu + \overleftarrow{\overline{\partial_\lambda}}\psi^\lambda\psi^\mu)(\overline{\partial}x^\nu + \overleftarrow{\overline{\partial_r}}\overline{\psi}^r\overline{\psi}^\nu) \\ &= \partial_\mu\partial_\nu\theta\partial x^\mu\overline{\partial}x^\nu, \end{aligned} \tag{23a}$$

$$T_F^{(1)} = \frac{1}{2}\partial_\mu\partial_\nu\theta\overline{\partial}x^\nu\psi^\mu, \tag{23b}$$

$$\overline{T}_F^{(1)} = \frac{1}{2}\partial_\mu\partial_\nu\theta\partial x^\nu\overline{\psi}^\mu \tag{23c}$$

with condition $\Box\theta = constant$, $\Box \equiv \partial^\mu\partial_\mu$. $\theta(x)$ in eq(23) is the background field corresponding to the singlet zero-norm state of eq(4). The induced "symmetry" is calculated to be

$$\phi \to \phi + \Box\theta \tag{24}$$

and

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \partial_\nu \theta. \tag{25}$$

Equation (25) is merely a change of gauge in the linearized graviton and can be absorbed to the symmetry of the linearized graviton. Equation (24) is the "symmetry" of the dilaton. The result that $\Box\theta$ is a constant is consistent with the fact that ϕ appears in the effecive equation of motion, constructed from vanishing $\sigma-$model $\beta-$function [13], in an overall factor $e^{-2\phi}$ other than differentiated. The interesting result here is that we identify the constant $\Box\theta$ to be the zero-norm state in equation (4). This completes the physical effects of all massless zero-norm states in the type II string spectrum. The "symmetries" presented in equations (21) and (24) were derived in the first order WFA. They can be broken in the higher order correction. However, if one tries to generalize the superconformal deformation to *second* order in the WFA, one immediately meets the difficulty of nonperturbative non-normalizibility of the 2d $\sigma-$ model, and is forced to introduce counterterms which consist of an infinite number of massive tensor fields. [14] This higher order effect is related to the stringy physics ($\alpha' \to \infty$) of string theory instead of point particle field theory. In fact, it was known that there exist important stringy bound states called (p,q) string which consists of p F-strings and q D-strings in the II B theory. [15] The coupling of axion χ to (p,q) string, which is an higher order effect and so can not be seen in our first order WFA, breaks the symmetry in equation (21) down to integer shifts. On the other hand, it was known that the symmetry in eq.(24) was broken down to the discrete $\Box\theta \equiv -2\langle\phi\rangle$ from the type II B supergravity. If we define

$$\rho = \chi + ie^{-\phi}, \langle\rho\rangle \equiv \frac{\theta}{2\pi} + \frac{i}{g_s} \equiv \tau, \tag{26}$$

these two discrete symmetries combine to form the well-known SL(2,Z) S-duality symmetry of II B string. Note that the nonlinearity of SL(2,Z) does not appear in our linear WFA. This is a generic feature of WFA in contract to the usual σ-model loop(α') expansion. The former

contains stringy phenomena (e. g. high energy symmetries) which can not be derived in the loop expansion scheme, while the latter is convenient to obtain the low energy effective field theory of the superstring. This will become clear when one considers the massive states of the string, which are crucial to make string theory different from the usual quantum field theory. An immediate application of this Type II B S-duality is the N=4, d=4 SUSY Yang-Mills S-duality [16], where dyon with the electric charge p and magnetic charge q can be interpreted as the end points of the (p,q) string on the D3-brane. The τ parameter in this SUSY gauge theory is defined to be

$$\tau = \frac{\theta_{YM}}{2\pi} + \frac{i}{g_{YM}^2}, \tag{27}$$

and is interpreted to be the constant ρ field of II B string in equation (26) associated to a stack of D3-branes.

IV. CONCLUSION

T-dual R-R zero-norm states motivate the introduction of D-branes into Type II string theory. They serve as symmetry charge parameters of R-R tensor forms. The study in this paper reveals again that all space-time symmetries, including the *discrete* T-duality and S-duality, are related to the zero-norm states in the spectrum. The unified description of S and T dualities makes one to speculate that they are all geometric symmetries (due to the redefinition of string backgrounds) and to conjecture the existence of a bigger discrete U-duality symmetry [9], [17] and its relation to the zero-norm state. In fact the SL(2,Z) S-duality of II B string led Vafa [18] to propose a 12d F-theory, where τ is the geometric complex structure modulus of torus T^2. One can even generalize this zero-norm state idea to construct new stringy massive symmetries of string theory. In particular, the existence of some massive R-R zero-norm states and other evidences make us speculate that string may carry some massive R-R charges [6]. Another interesting issue is the identification of D-brane charges with elements of K-theory groups. [19] How T-dual R-R zero-norm states relate to K-theory groups is an interesting question to study.

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